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Question Paper Code : 25139

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2018.

Third Semester

Computer Science and Engineering

MA 8351 — DISCRETE MATHEMATICS

(Common to Information Technology)

(Regulations 2017)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Construct the truth table for the following $P \wedge (P \vee Q)$.

2. Let $Q(x, y, z)$ denote the statement "$x + y = z$" defined on the universe of discourse Z, the set of all integers. What are the truth values of the propositions $Q(1,1,1)$ and $Q(1,1,2)$.

3. Show that in any group of 8 people at least two have birthdays which falls on same day of the week in any given year.

4. Solve $a_n - 5a_{n-1} + 6a_{n-2} = 0$.

5. An undirected graph G has 16 edges and all the vertices are of degree 2. Find the number of vertices?

6. Define incidence matrix of a simple graph.

7. Prove that in any group, identity element is the only idempotent element.

8. Let $f:(G,*) \to (G',\Delta)$ be a group homomorphism. Then prove that $[f(a)]^{-1} = f(a^{-1}), \forall a \in G$.

9. Define partial ordered set.

10. Determine whether D_8 is a Boolean algebra?

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the principle disjunctive normal form (PDNF) of $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$ without using truth table also find its Principle conjunctive normal form. (8)

(ii) Show that if x and y are integers and both xy and $x + y$ are even, then both x and y are even. (8)

Or

(b) (i) Show that $(P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R)) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology without using truth table. (8)

(ii) Show that the premises "A student in this class has not read the book" and "Everyone in this class passed the first examination" imply the conclusion "Someone who passed the first examination has not read the book". (8)

12. (a) (i) Prove by mathematical induction. (8)

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

(ii) Solve the recurrence relations $S(n) = S(n-1) + 2S(n-2)$ with $S(0) = 3, S(1) = 1; n \geq 2$ using generating function. (8)

Or

(b) (i) Find the number of integers between 1 to 100 that are not divisible by any of the integers 2,3,5 or 7. (8)

(ii) How many permutations can be made out of the letters of the word "Basic"? How many of these (8)

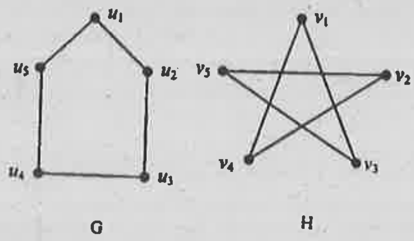
(1) Begin with B?

(2) End with C?

(3) B and C occupy the end places?

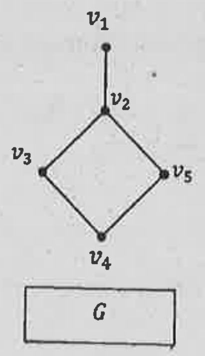
13. (a) (i) Prove that for a bipartite graph with n vertices has maximum of $\frac{n^2}{4}$ edges. (8)

(ii) Establish the isomorphic for the following graphs. (8)



Or

(b) (i) Define a subgraph. Find all the subgraphs of the following graph by deleting an edge. (8)



(ii) Prove that a connected graph has an Euler path if and only if and only if it has exactly two vertices of odd degree. (8)

14. (a) (i) Let $<S,*>$ be a semi group such that for $x, y \in S, x * x = y$, where $S = \{x, y\}$. Then prove that (8)

(1) $x * y = y * x$

(2) $y * y = y$

(ii) Find all the non-trivial subgroups of $(Z_{12}, +_{12})$. (8)

Or

(b) (i) Prove that $G = \{[1],[2],[3],[4]\}$ is an abelian group under multiplication modulo 5. (8)

(ii) Prove that intersection of two normal subgroups of a group G is again a normal subgroup of G. (8)

15. (a) (i) State and prove distributive inequalities in lattices. (8)

(ii) Prove that every chain is a distributive lattice. (8)

Or

(b) (i) Consider the set $D_{50} = \{1, 2, 5, 10, 25, 50\}$ and the relation divides (/) be a partial ordering relation on D_{50}. (8)

(1) Draw the Hasse diagram of D_{50} with relation divides.

(2) Determine all upper bounds of 5 and 10.

(3) Determine all lower bounds of 5 and 10.

(4) Determine LUB. of 5 and 10.

(5) Determine GLB. of 5 and 10.

(ii) State and prove De Morgan's laws in complemented and distributive lattice. (8)